short times are low ($\leq 150°F$), and little difference is seen between a 1% and a 2% error curve. For $\theta > 0.03$ sec, $2d = 0.01$ in. is appropriate. For the intermediate range, $0.001 < \theta < 0.03$ sec, either a 0.001-in. node can be used at an error between 1% and 2% (~5°F), or a node thickness between 0.001 and 0.01 in. can be used to hold the error near 2%. For $\theta > 5$ sec, a 0.1-in.-node thickness is appropriate.

Conclusions

Transient surface temperatures T_w for a flat plate can be calculated by an analytical method within chosen confidence limits, provided that a sufficient number of eigenvalues n are taken; correlations of N_{Bi}/N_{Fo} vs θ are presented to aid in the selection of the minimum n. A numerical method is useful for the more complex problems (variable material properties and boundary conditions) encountered in practice; correlations for predicting the correct node thickness to give an accurate T_w for a desired θ by this approach are based on $N_{Bi}\ \theta/N_{Fo}$, $h/k\alpha$, and $h/k\alpha^2$ for the large, short, and very short times, respectively.

References

[1] Schneider, P. J., *Conduction Heat Transfer* (Addison-Wesley Publishing Co., Inc., Cambridge, Mass., 1955), pp. 253 and 265.
[2] Chao, G. T. Y. and Jacobsen, J. A., "Hetran-computer program for transient heat transfer in nozzles," ABL/X-98, Allegany Ballistics Lab., Cumberland, Md. (May 1963).
[3] Goldsmith, A., Waterman, E., and Hirschhorn, H. J., *Thermophysical Properties of Solid Materials* (U. S. Air Force, Wright Air Development Division, 1960), Vol. 1, pp. 1-c-3d and 1-M-4.
[4] Anderson, J. T. and Botje, J. M., "Spatial truncation error analysis," Am. Soc. Mech. Engrs.—Am. Inst. Chem. Engrs. Paper 62-HT-27 (1962).
[5] Anderson, J. T. and Slonnegar, R. D., private communication (April 1963).

Theoretical vs Actual Nike-Apache Sounding Rocket Performance

D. W. Dembrow* and L. B. Jamieson†
NASA Goddard Space Flight Center, Greenbelt, Md.

The Nike-Apache vehicle (see Fig. 1) is a two-stage unguided sounding rocket, which has been recently added to the NASA family of sounding rockets. It is described in detail in Refs. 1 and 2. Nike-Apache, NASA No. 14.108 GI, was fired at Wallops Island, Virginia, on March 9, 1963. The vehicle was equipped with telemetry turnstile antennas and carried a 76-lb payload. The launcher was set at 108° azimuth and 75.7° elevation to correct for wind, the proposed launch angle being 80°.

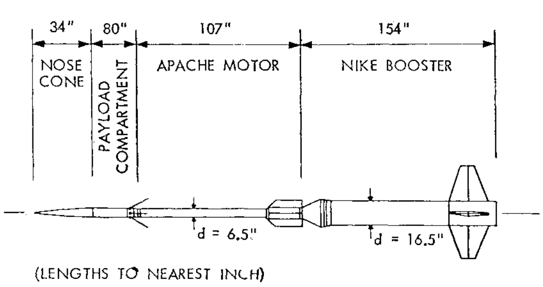

Fig. 1 Nike Apache.

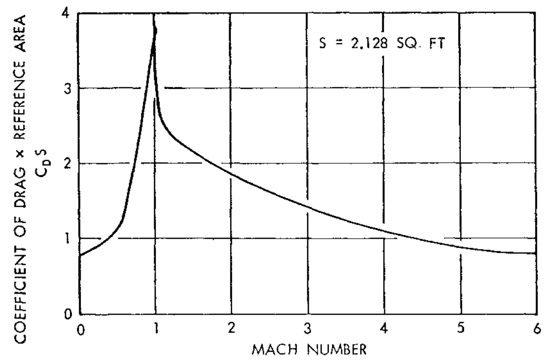

Fig. 2 C_DS vs Mach number, Nike, burning.

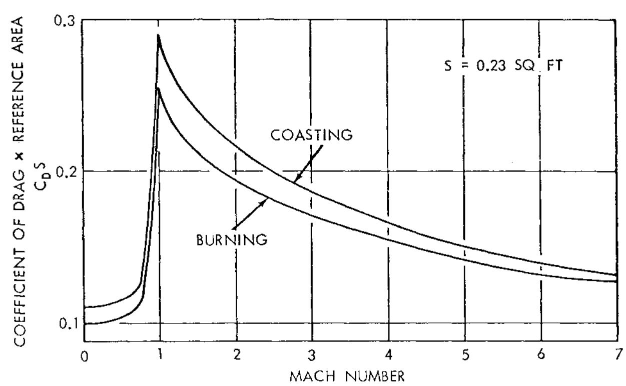

Fig. 3 C_DS vs Mach number, Apache with turnstile antenna, burning, and coasting.

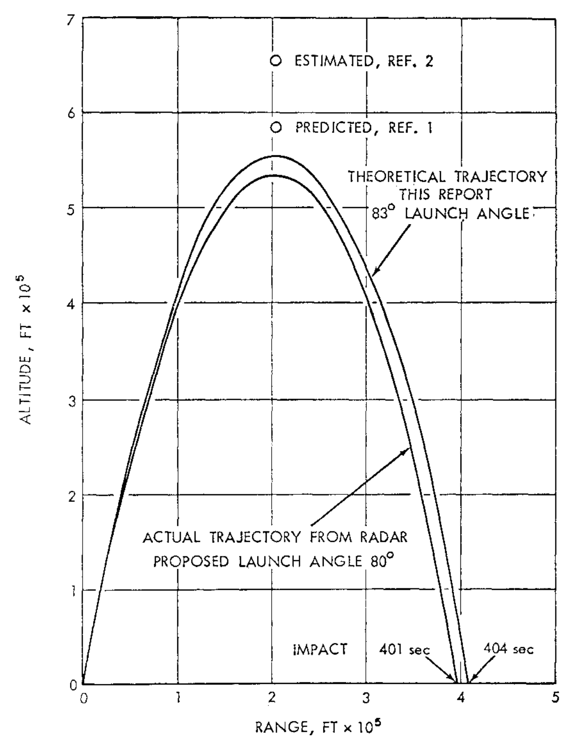

Fig. 4 Nike-Apache (14.108) trajectory.

The actual trajectory, as determined by smoothing measurements from two separate radar plots, was compared with theoretical trajectories computed for the Nike-Apache using the given payload, weights, thrusts, and drag conditions. This comparison indicates that the effective launch angle was slightly more than 83°. Corrections were made for the distance between the radar sites but not for the distance from

Received December 12, 1963; revision received January 17, 1964.

* Head, Signal Processing Section, Data Processing Branch, Data Systems Division. Member AIAA.
† Aerospace Technologist, Data Systems Division.

the radar sites to the launch site (about $\frac{1}{2}$ mile). The theoretical trajectory was determined by extrapolation from a series of computed trajectories having sea-level launch angles ranging from 76° to 82° in 2° intervals to obtain the 83° launch-angle equivalent. The theoretical trajectories were based on the equations of motion applying zero-lift, no-wind conditions for a spherical, nonrotating earth (i.e., weight, thrust, and drag). A program applying an n-stage two-dimensional point mass method was used on an IBM 7094 computer. The following conditions applied.

Phase 1: *Nike boost (from sea level), 3.5 sec*
Gross launch weight, 1611.5 lb
Constant mass flow rate and thrust
Drag during burning (see Fig. 2)
Separation at end of Nike burning

Phase 2: *Apache coasting, 16.5 sec*
Drag during coast phase (see Fig. 3)

Phase 3: *Apache burning, 6.4 sec*
Weight at second-stage ignition, 293.5 lb
Variable thrust, about 5000 lb
Drag during burning (see Fig. 3)

Phase 4: *Apache coasting*
Drag, same as phase 2
Atmospheric limit, 400,000 ft ARDC 1959

The results are shown in Fig. 4. It can be seen that the actual and theoretical trajectories disagree slightly. The peak altitude of the theoretical trajectory is higher than the actual by about $3\frac{1}{2}$ miles in about 100 miles. The results of other investigators who have computed theoretical trajectories for the Nike-Apache sounding rocket and their predicted altitudes are also shown in Fig. 4. The results from Ref. 1 would have predicted about 9 miles higher than the actual peak altitude. The results estimated from Ref. 2 (when adjusted for the additional drag of turnstile antennas) would be about 20 miles higher than the actual peak altitude.

References

[1] Jenkins, R., *Nike-Apache Performance Handbook*, NASA Goddard Space Flight Center X616-62-103 (July 1962).

[2] Russ, K. M., "Performance summary for the Nike-Apache sounding rocket vehicle," Rept. AST/E1R-00.93, Chance Vought Corp. under Contract NASA-1-1013, NASA Langley Research Center (March 1963).

Technical Comments

Hydrodynamic Impact of Conical-Nosed Vehicles during Vertical Water Entry

George Stoffmacher*
Douglas Aircraft Company, Santa Monica, Calif.

It is stated in Ref. 1 of the present comment that "because of their short durations, the impact loadings experienced by a vehicle during water entry will have little effect on the sub-surface trajectory." Upon assuming also that during the downward plunge the vehicle will be balanced on its nose inside the cavity and will continue along a vertical path, the authors present an equation of motion for the vehicle for zero cavitation number as follows:

$$W - \rho_W A h - \frac{\rho_W C_D A}{2g}\left(\frac{dh}{dt}\right)^2 = \frac{W}{g}\frac{d^2h}{dt^2} \tag{1}$$

where W, ρ_W, g, C_D, A, h, and t are, respectively, the weight, fluid density, acceleration of gravity, forebody drag coefficient, instantaneous cross-sectional area at the surface, depth of penetration below surface, and the time to penetrate a depth h. Then, upon integration of Eq. (1), they are able to arrive at a conservative estimate of the maximum depth to which the vehicle will travel.

If, however, one is interested in calculating the actual maximum force acting on the vehicle and its variation with time, one should not use the solution of Eq. (1). This solution will yield unconservative results, since the effect of impact has been neglected in the derivation of the equation of motion.

Received October 21, 1963.

* Structures Engineer, Advance Space Technology, Missile and Space Systems Division. Member AIAA.

This does not affect the total depth of penetration considerably, however, since only at instants in the neighborhood of $t = 0$ does the component of force due to impact dominate. However, since here it is desired to calculate the maximum force and its time history, this effect must be included.

This impact effect manifests itself in a so-called "virtual" mass of liquid which must be present as a consequence of the conservation of momentum. Reformulating Eq. (1) to include this effect leads to the following equation:

$$W - \rho_W A h - \frac{\rho_W C_D A}{2g}\left(\frac{dh}{dt}\right)^2 = \frac{d}{dt}(mv) = \frac{d}{dt}\left(m\frac{dh}{dt}\right) \tag{2}$$

where $m = (M_0 + M)$, and M is the induced virtual mass and M_0 is the original mass of the vehicle.

In the paper by Shiffman and Spencer,[2] an expression for the virtual mass M of the fluid is derived to be

$$M = k(\rho_W/g)h^3 \tan^3\beta \tag{3}$$

where k is the dimensionless "virtual mass" of the fluid and is a function of the semi-vertex angle β of the cone (see graph 1, Ref. 2). This is done by taking as a first approximation the

Table 1 Maximum force (number of g's)

1 Semi-vertex cone angle, deg	2 Stoff-macher	3 Shiffman & Spencer	4 Watanabe (experi-ments)	5 A	6 B
70	389	384	402	160	35.8
80	824	838	828	257	33.4
82.5	1105	1138	1145	330	32.7
85	1710	1735	1720	409	32.1